# A TEX-oriented Research Topic: Synthetic Analysis on Mathematical Expressions and Natural Language

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# A TEX-driven Life

- I met TEX when I was a high school student
  → at that time, I'm deeply interested in biology
- Later, I majored bioinformatics—combination of biology & informatics—for my bachelor degree
- I learned computer science with TEX

Implementing bioinformatics algorithms in TEX

## The Gotoh algorithm: DP

Sequence alignment has a slightly more complex scoring scheme.

**Example**

match = 1, mismatch = −1, $g(l) = -d - (l-1)e$

**The algorithm**

Sequence alignment in $O(mn)$ time:

$$M_{i+1,j+1} = \max\left\{M_{ij}, I_{x_{ij}}, I_{y_{ij}}\right\} + c_{a_i b_j}$$

where

$$I_{x_{i+1,j}} = \max\left\{M_{ij} - d, I_{x_{ij}} - e, I_{y_{ij}} - d\right\},$$
$$I_{y_{i,j+1}} = \max\left\{M_{ij} - d, I_{y_{ij}} - e\right\}.$$

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Implementing bioinformatics algorithms in TEX

## The **Gotoh** package

**Usage**

- `\Gotoh`{⟨*sequence A*⟩}{⟨*sequence B*⟩}
  - Executes the algorithm
  - Returns the results to specified CSs
- `\GotohConfig`{⟨*key-value list*⟩}
  - Setting various parameters
  - e.g. algorithm parameters, CSs to store results

**Example**

Input:

```
\Gotoh{ATCGGCGCACGGGGGA}
      {TTCCGCCCACA}
\texttt{\GotohResultA} \\
\texttt{\GotohResultB}
```

Output:

```
ATCGGCGCACGGGGGA
TTCCGCCCAC.....A
```

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# An Idea from TEX: Toward NLP

## Representing meanings with TEX macros

Instead of directly using primitives or standard commands, we can define our own macros which reflect "meanings".

## Example

To express a vector with a **bold** font:

- ✗ Directly writing "`$\mathbf{x}$`"
- ✓ Defining "`\def\vector#1{\mathbf{#1}}`" and using the macro as "`$\vector{x}$`"

But: many authors neglect such representation.

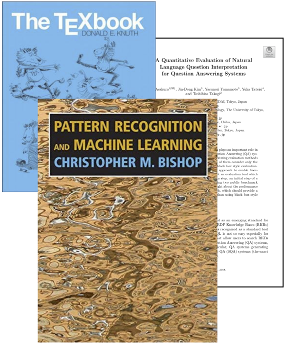
How about automating the process?

# Targets: STEM Documents

The targets of our work are Science, Technology, Engineering, and Mathematics (STEM) documents.

Example

- Papers,
- Textbooks, and
- Manuals, etc.




STEM documents are:

- essence of human knowledge
- well organized (semi-structured)
- texts with mathematical expressions

# Long-term Goal: Converting STEM Documents to Formal Expressions

**STEM Documents (Natural Language + Formulae)**

Papers, textbooks, manuals, etc.

↓ Conversion

**Computational Form (Formal Language)**

Executable code, first-order logic, etc.

The conversion enables us to:

- construct databases of mathematical knowledge
- search for formulae

# Necessity of Synthetic Analysis

## Interaction among texts and formulae

Texts and formulae are complimentary to each other: [Kohlhase and Iancu, 2015]

- Texts explains formulae (and vice versa)
- Texts in formulae E.g. $\{x \in \mathbb{N} \mid x \text{ is prime}\}$
- Notations and verbalizations
  E.g. $1 + 2$ and “one plus two”

Deep synthetic analyses on natural language and mathematical expressions are necessary.

# Grounding Elements to Mathematical Objects

- Elements in formulae and their combination can refer to mathematical objects
- The detection is fundamental for understanding STEM documents

## Example

For example, $x$ might describe the outcome of flipping a coin, with $x = 1$ representing ‘heads’, and $x = 0$ representing ‘tails’. We can imagine that this is a damaged coin so that the probability of landing heads is not necessarily the same as that of landing tails. The probability of $x = 1$ will be denoted by the parameter $\mu$. The probability distribution over $x$ can therefore be written in the form

The probability of ‘heads’ on top, float, $0 \leq \mu \leq 1$

$$\mathrm{Bern}(x \mid \mu) = \mu^x (1-\mu)^{1-x}$$

The result of coin flipping, int, $x \in \{0, 1\}$

which is known as the *Bernoulli* distribution. (PRML, pp. 86–87)

# Difficulty of the Grounding

Factors which make the detection highly challenging:

- ambiguity of elements (see below)
- syntactic ambiguity of formulae E.g. $f(a+b)$
- necessity for common sence & domain knowledge
- severe abbreviation

**Usage of character y in the first chapter of PRML (except exercises)**

| Text fragment from PRML Chap. 1 | Meaning of $\mathbf{y}$ |
|---|---|
| ... can be expressed as a function $\underline{\mathbf{y}}(\mathbf{x})$ ... | a function which takes an image as input |
| ... an output vector $\underline{\mathbf{y}}$, encoded in ... | an output vector of function $\mathbf{y}(\mathbf{x})$ |
| ... two vectors of random variables $\mathbf{x}$ and $\underline{\mathbf{y}}$ ... | a vector of random variables |
| Suppose we have a joint distribution $p(\mathbf{x}, \underline{\mathbf{y}})$ ... | a part of pairs of values, corresponding to $\mathbf{x}$ |

# Semantics Over Natural Language and Mathematical Expressions

There are ambiguity arise only when context exists. For instance, “equals signs” (=) in formulae have at least three meanings: definition, identity, and equation.

## Example

Let $a = 4$, $b = 3$. Suppose we have to solve

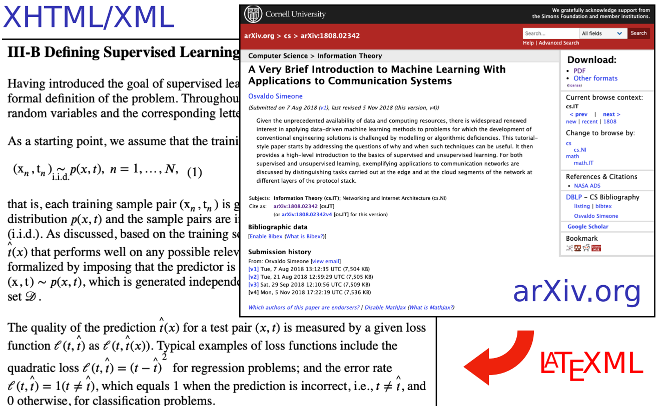
$$ax^4 + bx^2 + 1 = 0.$$

To reach the answer, “difference of two” is helpful:

$$p^2 - q^2 = (p+q)(p-q).$$

# Dataset arXMLiv

- papers from arXiv in XML format [Ginev+, 2009]
  - converted from LaTeX via LaTeXML
  - formulae are in MathML markups



# A Little Note for MathML

- a W3C Recommendation [Ausbrooks+, 2014]
- includes two markups: presentation and content

## Presentation Markup

This shows syntax:

```
<msup>
  <mfenced>
    <mi>a</mi>
    <mo>+</mo>
    <mi>b</mi>
  </mfenced>
  <mm>2</mm>
</msup>
```

## Content Markup

This shows semantics:

```
<apply>
  <power>
  <apply>
    <plus/>
    <ci>a</ci>
    <ci>b</ci>
  </apply>
  <cn>2</cn>
</apply>
```

$$(a+b)^2$$

# The Research Plan

## Creating a dataset (pilot annotation)

- do the grounding by hand for some papers in arXiv
  → Let me show you a demonstration
- I would also like to do it for some textbooks

## Automating the detectiion

Combination of rule-based and machine learning with features such as:

- apposition nouns E.g. “a function $f$”
- syntactic information in formulae
  E.g. does it appear inside an argument or not?
- distance from the former appearence

# Possible Applications

- Mathematical Information Retrieval (MIR)
  → enables us to create scientific knowledge bases
- Automatic code generation   E.g. Python, Coq, etc.
- Searching for mathematical expressions

Example

Let us think about searching for:

$$x^n + y^n = z^n \quad (n \geq 3).$$

It is easy to search if you know a keyword *Fermat's Last Theorem*, but otherwise...

# Conclusions

- converting STEM documents to computational form is beneficial and challenging
- for the conversion, synthetic analysis on natural language and mathematical expressions is required
- Currenly, we are working on creating a dataset
- Possible applications: MIR, code generation, searching for formulae

TEX has a power to change one's life!