The exact placement of superscripts and subscripts

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Introduction

Consider the following segment of mathematics in "uncramped" display (D) style.

$$\theta^{At}\gamma_{jk}$$

It is conceivable that an author would want the At term to be higher on the θ, since the base of the At is apparently all the way down to the midline of the θ. Such higher placement might make room for different levels of superscripts, like those used in tensor notation. This need commonly occurs when the nucleus, in this case θ, has zero depth and a height that is greater than its width. That same author might also want the jk to be lower on the γ, since, on quick inspection, the jk might appear to be on the same level as the γ (making it a product of three terms). This situation commonly occurs when the nucleus has a descender and the subscript has an ascender (and where a significantly large nucleus depth exists compared to the ascender height).

In both these cases, it would be impractical for the author to address these issues by changing to a completely different font that has more appealing font dimensions, or to use a specially designed character, when its presence in other contexts would have just as many problems as before.

As is usually the case with TEX, there is a way to address these issues with a scalpel and not a club. Individual elements in a mathematical expression may be precisely placed to an author's exact requirements through the careful manipulation of local font dimensions, without the need of creating new fonts or using custom character glyphs.

The placement of superscripts and subscripts when generating boxes from formulas follows the rules set forth in Appendix G of *The TEXbook*. In particular, Rules 18a, 18c, and 18d apply to the first situation, where there is a superscript on θ but not a subscript, and Rules 18a and 18b apply to the second situation, where there is a subscript on γ but not a superscript. Other combinations of the rules apply when there is both a superscript and a subscript on the same nucleus.

Superscript placement

Consider the first situation, where an author wishes to raise the placement of the At term slightly higher. The standard placement rules under this circumstance are as follows.

18a. Since the nucleus is a character box, set $u = 0$.

18c. Set box X to the contents of the superscript, here At, in scriptstyle, and add `\scriptspace` to the width of X. Then set

$$u \leftarrow \max\left\{u, \sigma_{13}, d\,(\theta) + \frac{1}{4}\,|\sigma_5|\right\},$$

since we are in display style, where $d\,(\theta)$ is the depth of the θ character, σ_{13} is font dimension 13 of `\textfont2` (the so-called "sup1" dimension), and σ_5 is font dimension 5 of `\textfont2` (the so-called "x-height"). Note that $|\sigma_5|$ is used since the x-height of a font may (conceivably) be negative.

18d. Append box X to the horizontal list, shifting it up by u.

These simple steps show that the only way to influence the placement of the superscript box X (without changing the dimensions of the nucleus itself) is to temporarily change the 5^{th} and/or 13^{th} font dimension of `\textfont2`. The most direct way of doing so is to increase σ_{13} by an amount that exceeds $d\,(\theta) + \frac{1}{4}\,|\sigma_5|$. For Plain TEX, `\textfont2` is `\tensy` (an alias for `cmsy10`), and $d\,(\theta) = 0$, $\sigma_5 = 4.30554$ pt, and $\sigma_{13} = 4.12892$ pt. This means $u = \sigma_{13}$ when θ^{At} is translated under Plain TEX into a horizontal list. However, the height of θ is 6.94444 pt (which shows that the superscript is normally placed $\frac{4.12892}{6.94444} \approx 59.456\%$ of the way up θ from the baseline), so that $\sigma_{13} = 7$ pt would place the baseline of At less than $\frac{1}{10}$ pt above the top of the θ. Comparing these two options, we have

$$\theta^{At} \quad \text{versus} \quad \theta^{At}$$

which was produced by the following TEX code.

```
\tmp=\the\fontdimen14\tensy
\centerline{$\theta^{At}$\quad
   {\rm versus}\quad
   $\fontdimen14\tensy=7pt
   \theta^{At}$}
% See note below about the
% use of font dimension 14.
\fontdimen14\tensy=\tmp
```

Since font dimension changes are, by default, global, any such changes must be explicitly reversed. This is the purpose of the `\tmp` dimension register in the previous example. Note also that it is the font dimensions of `\textfont2` that are changed to produce the desired results, and not the dimensions

of \scriptfont2, even though the placement of the "script" parts are affected.

For expressions that have only superscripts, in regular math mode, i.e., not in display style, the 14^{th} font dimension (sup2) of \textfont2 would be changed (instead of the 13^{th} one as before), and in "cramped" display style, the 15^{th} font dimension (sup3) of \textfont2 would be changed.

Subscript placement

The procedure for subscript placement is similar to that of superscript placement, except that (a) it is slightly more straightforward, except (b) it uses a "cramped" style. To wit:

18a. Since the nucleus is a character box, set $v = 0$.

18b. Set box X to the contents of the subscript, here jk, in "cramped" scriptstyle, and add \scriptspace to the width of X. Append box X to the horizontal list, shifting it down by

$$\max\left\{v, \sigma_{16}, h\left(\gamma\right) - \frac{4}{5}\left|\sigma_5\right|\right\},$$

since we are in display style, where $h\left(\gamma\right)$ is the height of the γ character, σ_{16} is font dimension 16 of \textfont2 (the so-called "sub1" dimension), and σ_5 is font dimension 5 of \textfont2 (the so-called "x-height"). Note that $|\sigma_5|$ is used since the x-height of a font may (conceivably) be negative.

We have $h\left(\gamma\right) = 4.30554$ pt, $\sigma_5 = 4.30554$ pt, and $\sigma_{16} = 1.49998$ pt. This means the subscript is lowered by $\sigma_{16} = 1.49998$ pt under Plain TEX. However, the depth of γ is 1.94444 pt, and the height of k in scriptstyle is 4.8611 pt. This means $\sigma_{16} = (1.94444 + 4.8611)$ pt $= 6.8055$ pt would place the top of jk at the bottom of the γ. Comparing these two options, we have

$$\gamma_{jk} \quad \text{versus} \quad \gamma_{\,jk}$$

which was produced by the following TEX code.

```
\tmp=\the\fontdimen16\tensy
\centerline{$\gamma_{jk}$\quad
   {\rm versus}\quad
   $\fontdimen16\tensy=6.8055pt
   \gamma_{jk}$}
\fontdimen16\tensy=\tmp
```

Note that for expressions that have only subscripts, the 16^{th} font dimension of \textfont2 would be changed regardless of the style where the expression occurs.

Optional changes

Additional aesthetic changes for the second example might include moving the subscripts slightly closer to the γ due to the shape of the j. This may be accomplished by a small kern in the subscript, such as

```
\gamma_{\mkern-2.7mu jk}
```

However, considering how small the differences may be relative to the font dimensions, it may be difficult to distinguish the horizontally unadjusted version with the adjusted version.

$$\gamma_{jk} \quad \text{versus} \quad \gamma_{\,jk}$$

Both types of scripts together

Rule 18a contains more complicated instructions when the nucleus of the expression is not simply a character, and Rules 18d, 18e, and 18f contain even more complicated instructions when there is a superscript *and* a subscript in the same expression. Taken collectively, the effect of this latter set of rules is not the same as simply placing the superscript independently of the subscript, one after the other, in either order. These two more challenging circumstances are not covered here; everything an author needs to know, and a lot more, to make arbitrary changes in superscript and subscript placements, is in Appendix G of *The TEXbook*. It suffices to state that no matter how exotic the expression becomes, the placement of superscripts and subscripts in TEX is enormously flexible due to the ability to change a few font dimensions at strategic positions.

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